
MATHEMATIK FÜR MEDIENWISSENSCHAFTLER/INNEN

Zusatzaufgaben

Aufgabe 1 Zeigen Sie, dass gilt:

- a) $\frac{\sin^3(x) + \cos^2(x) \sin(x)}{\cos(x)} = \tan(x)$
- b) $\frac{2x + x^2}{x^3 - 4x} = \frac{1}{x - 2}$
- c) $x(\ln(2x) + \ln(\frac{1}{2x}) - 5) = -5x$
- d) $\frac{e^{2x} + 3e^x + 2}{e^x + 2} = e^x + 1$
- e) $1 + \tan(2x) = \frac{(1 - 2\sin^2(x)) + 2\sin(x)\cos(x)}{\cos^2(x) - \sin^2(x)}$
- f) $\frac{e^{2x} + e^x}{e^x} = e^x + 1$
- g) $\sqrt{\cos^3(x) + \cos(x)\sin^2(x)} = \sqrt{\cos(x)}$
- h) $\frac{b^{2n}a^n - a^n}{a^n + b^n a^n} = b^n - 1$
- i) $\cos(3x) = \cos^3(x) - 3\sin^2(x)\cos(x)$
- j) $2\cos(2x) = 2 - 4\sin^2(x)$

Lösung:

a)

$$\begin{aligned}\frac{\sin^3(x) + \cos^2(x)\sin(x)}{\cos(x)} &= \frac{\sin(x)}{\cos(x)} (\sin^2(x) + \cos^2(x)) \\ &= \tan(x)\end{aligned}$$

b)

$$\begin{aligned}\frac{2x + x^2}{x^3 - 4x} &= \frac{x(2 + x)}{x(x^2 - 4)} \\ &= \frac{2 + x}{(x + 2)(x - 2)} \\ &= \frac{1}{x - 2}\end{aligned}$$

c)

$$\begin{aligned}x(\ln(2x) + \ln\left(\frac{1}{2x}\right) - 5) &= x(\ln\left(2x \cdot \frac{1}{2x}\right) - 5) \\ &= x \ln(1) - 5x \\ &= 0 - 5x \\ &= -5x\end{aligned}$$

d)

$$\begin{aligned}\frac{e^{2x} + 3e^x + 2}{e^x + 2} &= \frac{(e^x + 1)(e^x + 2)}{e^x + 2} \\ &= e^x + 1\end{aligned}$$

e)

$$\begin{aligned}1 + \tan(2x) &= 1 + \frac{\sin(2x)}{\cos(2x)} = 1 + \frac{2 \sin(x) \cos(x)}{\cos^2(x) - \sin^2(x)} \\ &= \frac{(\cos^2(x) - \sin^2(x)) + 2 \sin(x) \cos(x)}{\cos^2(x) - \sin^2(x)} \\ &= \frac{(1 - 2 \sin^2(x)) + 2 \sin(x) \cos(x)}{\cos^2(x) - \sin^2(x)}\end{aligned}$$

f)

$$\begin{aligned}\frac{e^{2x} + e^x}{e^x} &= \frac{e^{2x}}{e^x} + \frac{e^x}{e^x} \\ &= e^x + 1\end{aligned}$$

g)

$$\begin{aligned}\sqrt{\cos^3(x) + \cos(x) \sin^2(x)} &= \sqrt{\cos(x)(\cos^2(x) + \sin^2(x))} \\ &= \sqrt{\cos(x)}\end{aligned}$$

h)

$$\begin{aligned}\frac{b^{2n}a^n - a^n}{a^n + b^na^n} &= \frac{a^n(b^{2n} - 1)}{a^n(1 + b^n)} \\ &= \frac{(b^{2n} - 1)}{(1 + b^n)} \\ &= \frac{(b^n + 1)(b^n - 1)}{(1 + b^n)} \\ &= b^n - 1\end{aligned}$$

i)

$$\begin{aligned}\cos^3(x) &= \cos(2x + x) \\ &= \cos(2x) \cos(x) - \sin(2x) \sin(x) \\ &= (\cos^2(x) - \sin^2(x)) \cos(x) - (2 \sin(x) \cos(x)) \sin(x) \\ &= \cos^3(x) - \sin^2(x) \cos(x) - 2 \sin^2(x) \cos(x) \\ &= \cos^3(x) - 3 \sin^2(x) \cos(x)\end{aligned}$$

j)

$$\begin{aligned}2 \cos(2x) &= 2(\cos^2(x) - \sin^2(x)) \\ &= 2((1 - \sin^2(x)) - \sin^2(x)) \\ &= 2(1 - 2 \sin^2(x)) \\ &= 2 - 4 \sin^2(x)\end{aligned}$$